

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT1

Branch: B. Tech (All)

Semester : 1

Date : 26/02/2020

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) If $y = \cos \theta + i \sin \theta$, then the value of $y + \frac{1}{y}$ is
 (A) $2 \cos \theta$ (B) $2 \sin \theta$ (C) $2 \operatorname{cosec} \theta$ (D) $2 \tan \theta$
- b) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then
 (A) $a = 2, b = -1$ (B) $a = 1, b = 0$ (C) $a = 0, b = 1$ (D) $a = -1, b = 2$
- c) If $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is given to be continuous at $x = 0$, then the value of $f(0)$ must be
 (A) $a + b$ (B) $a - b$ (C) $b - a$ (D) $\log\left(\frac{a}{b}\right)$
- d) $\lim_{x \rightarrow \infty} x^n e^{-x} = \underline{\hspace{2cm}}$
 (A) 0 (B) 1 (C) 2 (D) none of these
- e) The interval of convergence of the logarithmic series
 $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty$ is
 (A) $-1 < x \leq 1$ (B) $-1 < x < 2$ (C) $-\infty < x < \infty$ (D) $-1 \leq x \leq 1$
- f) The sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is
 (A) $\log 2$ (B) zero (C) infinite (D) none of these
- g) The tangents at the origin are obtained by equating to zero
 (A) the lowest degree terms (B) the highest degree terms
 (C) constant term (D) none of these
- h) If the power of y are even, then the curve is symmetrical about
 (A) X-axis (B) Y-axis (C) about both X and Y axes
 (D) none of these



- i) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \infty$ represent expansion of
 (A) $\sinh x$ (B) $\cosh x$ (C) $\cos x$ (D) e^x
- j) If $y = \sin^{-1} x$, then x equal to
 (A) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (B) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$
 (C) $1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \dots$ (D) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$
- k) If $u = ax^2 + 2hxy + by^2$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to
 (A) $2u$ (B) u (C) 0 (D) none of these
- l) If $f_1 = \frac{vw}{u}$, $f_2 = \frac{wu}{v}$, $f_3 = \frac{uv}{w}$; then $\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)}$ is equal to
 (A) 0 (B) 1 (C) 3 (D) none of these
- m) If $Q = r \cot \theta$, then $\frac{\partial Q}{\partial r}$ is equal to
 (A) $\cot \theta$ (B) $-\cos ec^2 \theta$ (C) $\cot \theta - r \cos ec^2 \theta$ (D) $\frac{1}{2} \cot \theta$
- n) If $u(x, y, z) = 0$ then the value of $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$ is equal to
 (A) 1 (B) -1 (C) 0 (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) Find the continued product of all the values of $\left(\frac{1}{2} + i \frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. (5)
- b) Evaluate: $\lim_{x \rightarrow 0} (a^x + x)^{\frac{1}{x}}$ (5)
- c) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a}{x} - \cot \frac{x}{a}\right)$ (4)

Q-3 Attempt all questions (14)

- a) Using De Moivre's theorem prove that (5)
 (i) $\cos 5\theta = 5 \cos \theta - 20 \cos^3 \theta + 16 \cos^5 \theta$
 (ii) $\sin 5\theta = 5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta$
- b) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ (5)
- c) Prove that $\sec h^{-1}(\sin \theta) = \log \cot \frac{\theta}{2}$. (4)

Q-4 Attempt all questions (14)

- a) Prove that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$ (5)



b) Expand $\tan^{-1} x$ up to the first four terms by Maclaurin's series. (5)

c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$. (4)

Q-5 Attempt all questions (14)

a) Examine the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots + \frac{x^n}{n^2 + 1} + \dots$ for convergence using ratio test. (5)

b) Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$ using comparison test. (5)

c) Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. (4)

Q-6 Attempt all questions (14)

a) If $u = \tan^{-1} \left(\frac{x^2 + y^2}{x - y} \right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$. (5)

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that (5)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x + y + z)^2}.$$

c) Using Sandwich theorem prove that (4)

$$(i) \lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0 \quad (ii) \lim_{x \rightarrow 0} x^2 \cos 20\pi x = 0$$

Q-7 Attempt all questions (14)

a) Trace the curve $r = a(1 + \cos \theta)$. (5)

b) If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. (5)

c) Using the formula $R = \frac{E}{I}$, find the maximum error and percentage of error in R if $I = 20$ with a possible error of 0.1 and $E = 120$ with a possible error of 0.05 and $R = 6$. (4)

Q-8 Attempt all questions (14)

a) Trace the curve $xy^2 = 4a^2(2a - x)$. (5)

b) Discuss the maxima and minima of $xy + 27\left(\frac{1}{x} + \frac{1}{y}\right)$. (5)

c) Discuss the continuity of the function (4)

$$f(x, y) = \frac{x}{\sqrt{x^2 + y^2}} \text{ when } (x, y) \neq (0, 0) \text{ and}$$

$$f(x, y) = 2 \text{ when } (x, y) = (0, 0)$$

